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MODELING WATER BALANCE OF DAMMED LAKES USING COMPUTER CODE MATLAB-SIMULINK®

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Abstract

The paper presents a water balance of a flow-through, dammed lake, consisted of the following terms: surface inflow, underground inflow/outflow based on the Dupuit's equation, precipitation on the lake surface, evaporation from water surface and outflow from the lake at which a damming weir is located. The balance equation was implemented Matlab-Simulink®. Applicability of the model was assessed on the example of the Sławianowskie lake of surface area 276 ha and mean depth - 6.6 m, Water balances, performed for month time intervals in the hydrological year 2009, showed good agreement for the first three months only. It is concluded that the balancing time interval should be shorter (1 day) to minimize the errors. For calibration purposes, measurements of ground water levels in the vicinity of the lake are also recommended.

Keywords: water balance, dammed lake, Matlab-Simulink, Sławianowskie Lake

1. INTRODUCTION

Poland is one of countries of scarce water resources, thus any form of water storage is appreciated. One of the cheapest forms of storage is damming lakes, but it should be done carefully to avoid environmental losses and inundations. Recent assessment has shown that 60% of Polish lakes is in unsatisfactory ecological state. The problem is that damming is a form of morphological deterioration (breakdown of free fish passage), but on the other hand it helps to keep a better lake water quality thanks to a lower water temperature during

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summertime (better oxygenation) and better dilution of pollutants [9]. To make an active water management on a dammed lake one has to know the priorities set by the stakeholders [8, 15]. According to the Warta River Basin Management Plan [14], the priorities of water authorities are as follows:

a. drinking water supply,
b. production of food and pharmaceuticals,
c. fulfillment of environmental goals,
d. navigation and justified industrial purposes including irrigation,
e. tourism and recreation.

One of the basic tools in a lake water management, including optimization [6, 15], is a reliable model of its water balance. Such models were implemented in specialized codes: DYNHYD/WASP [16], MODFLOW [7], MIKE SHE [4] which are sometime very complex and hard to operate. Among simpler computer programs are those based on Matlab-Simulink®. There are also similar free and open source software called Octave and Scilab, which can be applied instead of the commercial Matlab-Simulink®**.**

Giusti et al. [3] have shown that the last code can be successfully applied for modeling lake water balance, however they considered the underground outflow (ignoring underground inflow), using three tuned parameters (constants) α, β and γ as follows:

$$Q_{ou} = -\alpha \sqrt{|h-H|} \; (h > H) \tag{1.1}$$

where: h - water depth in the lake, measured from the mean bottom level, and
H - groundwater (water table) depth, calculated as:

$$H = -\beta - \gamma \; f(d) \tag{1.2}$$

where: d - a successive day of the year.

This model of underground flow seems to be physically unjustified and the calibration procedure using it is rather complex and time-consuming.

The purpose of this paper is to elaborate a simplified model of lake water balance embracing underground inflow and outflow, described by the Dupuit's equation, and to discuss its applicability for prediction of storage in dammed lakes.

2. MODEL FORMULATION

2.1. Lake water balance equation

One of the most important tools applied for a dammed lake water management is the lake water balance with time interval: 1 month, 1 decade or 1 day, depending on variability of the meteorological and hydrological conditions. For a dammed lake without ice cover its daily water balance in the ***j***-th day can be expressed as:

$$\frac{\Delta h_j}{\Delta t} = P_j - E_j + H_{is\ j} - H_{os\ j} + H_{iu\ j} \tag{2.1}$$

where: Δh_j - water level (storage) change in Δt = 1 d, mm, P_j - precipitation on the lake surface, mm d^{-1}, E_j - evaporation from the lake surface in the j-th day , mm d^{-1}, $H_{is\ j}$ - surface runoff from the catchment, mm d^{-1}, $H_{os\ j}$ - outflow from the lake at the damming section, mm d^{-1}, $H_{iu\ j}$ - underground inflow or outflow (negative inflow), mm d^{-1}.

Below all elements of the above presented water balance are shortly described.

2.2. Precipitation on the lake surface

Precipitation should be measured closely to the lake as well as in its catchment. Long-term balances (with decade’s or month’s time intervals) can be based on records taken from remote (up to 30-50 km) stations. Heavy rains, accompanying summer storms, are local phenomena and due to their large spatial variability they can not be accurately estimated using records form such stations.

Snow melting is rather hard to forecast but it may change significantly the lake water balance during winter and early spring months.

2.3. Evaporation from the lake surface

There are several methods of evaporation estimation. One of the simplest and most popular is the Penman method [1, 2]. The rate of evaporation from the lake surface according to Penman [1, 11] is:

$$E_j = 0{,}36\,\delta\left(1 + 0{,}5 \cdot WS_2\right) \cdot n, \text{ mm month}^{-1} \tag{2.2}$$

where δ - air vapor pressure deficit in hPa, calculated as:

$$\delta = E_s - e = E_s - E_s RH/100 = E_s\left(1 - 0{,}01\,RH\right) \tag{2.3}$$

E_s - saturation partial pressure of water vapor for the air temperature T:

$$E_s = 6{,}112 \exp\left[17{,}62\, T/\left(243{,}12 + T\right)\right] \tag{2.4}$$

e - mean monthly partial pressure of water vapor in the free air stream at 2,0 m over the lake surface, hPa, RH - relative humidity of the air above the lake surface, %, WS_2 - wind speed at 2,0 m over the lake surface, m s^{-1}, T - average monthly air temperature, ^{o}C, n - number of days in a month.

Knowing the last three variables (RH, WS_2 and T) recorded by majority of meteorological stations, one can easily solve the Eqs. (2.4), (2.3) and (2.2). There is also a methodology of estimation of lake evaporation from the lake energy balance, but it is not covered by our paper.
The lake cover of vegetation and ice cover were ignored in this preliminary analysis.

2.4. Stream inflow/outflow and diffuse runoff

Surface inflow from the direct lake catchment is called a diffuse runoff. It depends on soils, their infiltration properties, slopes, cover roughness etc. It can be estimated roughly using computer codes like Stormwater Management Model [13], but in our analysis it was included into underground inflow.
Stream inflows can be measured and/or estimated using a precipitation/runoff model. Stream outflows can also be measured directly or indirectly, e.g. using drop structures. For a typical weir with a sluice-gate the following formula can be applied:

$$H_{os\,j} = \frac{86400 \cdot Q_j}{A} = \frac{C \cdot z \cdot \sqrt{h-a}}{A} \quad [m^3 / m^2 \cdot d] \tag{2.5}$$

Q - mean flow discharge during the time interval, dm^3s^{-1}, $A(h)$ - lake water surface area, m^2, C - constant, dependent on discharge coefficient, acceleration due to gravity and weir width, $m^{1.5}d^{-1}$, z - sluice gate opening height, m, a - difference between the weir invert level and the mean lake bottom level, m.

For weirs with a spillway the flow discharge is proportional to the height of head of water over the crest (h - y) powered to 1.5, where y is the crest level elevation above the mean lake bottom level, m. The total monthly inflow and outflow was calculated as the sum of daily inflows or outflows.

2.5. Underground inflow/outflow

Underground feeding is in many cases very important, but it is hard to model due to a complex lithological structure of the aquifers surrounding a given lake.

It is usually calculated from a water balance equation, knowing lake water levels [10], thus that method can not be used for storage prediction.

In the simplest case of uniformly distributed permeable soils, it is hypothesized that to calculate the unit discharge of the underground inflow, the Dupuit's formula can be applied in the form:

$$q = \frac{k\,(H^2 - h^2)}{2\,L} \quad [m^3 / m \cdot d] \tag{2.6}$$

Therefore, taking into account the whole lake perimeter (bank line) one can write:

$$H_{iu} = 1000\,\frac{q \cdot L_B}{A} \cdot n \quad [\,mm / month\,] \tag{2.7}$$

where: k - unknown parameter (averaged hydraulic conductivity), $m{\cdot}d^{-1}$, H - piezometric head at the distance L (depends on soil permeability) along a transect, above the mean bottom level of the lake, m, $L_B = f(h)$ - length of the bank line boundary, m, $A = f(h)$ - lake water surface area, m^2; e.g. $A = A_{max}(h/h_{max})^{2/p}$ where: p - parameter representing the submerging bank slope profile.

When piezometric measurements are not available, one can take - as a rough approximation - the H values as the yearly mean water depth h or the lake depth h at the end of the previous, sufficiently long time period (e.g. month) or to treat H, L and k like three unknown parameters in a similar way like the Giusti's et al [3] approach (see Eqs. (1.1) and (1.2)) or two parameters: H and L/k, only.

3. MODEL CALIBRATION

Applicability of the model was assessed on the example of the Sławianowskie lake (Krajeńskie Lake District - see Fig. 1). This lake has been an object of detailed investigations by the Poznan department of the Institute of Meteorology and Water Management since the year 2007. Surface area of the lake is 276 ha, mean depth - 6.6 m, the maximum depth - 15.0 m and the length of bank line L_B = 21625 m [5]. The lake was divided in two basins (segments): the shallower one with a mean depth of 5.0 m consisted of three parts (bays) and a deeper one, in the central part of the lake, with a mean depth of 7.7 m. Several ditches and small river Kocunia are inflowing to the lake. There are gauge stations at the inlet (village Wiktorówko) and at the outlet (village Buntowo), where water levels (every day) and flows (temporarily) are measured. The lake catchment area, i.e. the inflowing river catchment, is 104 km^2. Precipitation data have been gathered from the closest meteorological stations in Krajenka and Wysoka. Data on relative air humidity, air temperatures as well as wind speed to calculate evaporation were taken from the station in Piła. The values calculated from Eq. (2.2) have occurred to low, therefore they were increased by 50% to cope with the measurements made in the IMGW evaporation station in Piła. The water balance was analyzed for the whole lake with a mean depth and for the above mentioned two basins with their mean depths. Due to small slopes of the surrounding valley and good infiltration conditions its diffuse runoff was included in the underground inflow. Water balances were performed for month time intervals in the hydrological year 2009.

As a goodness-of-fit criterion an absolute error of water depth in the lake at the end of each month was applied and a mean yearly absolute error was adopted in the form:

$$U_{\Delta h} = \frac{1}{12}\sum_{j=1}^{12}\left|\Delta h_{cj} - \Delta h_{mj}\right| \tag{3.1}$$

where: $\Delta\, h_c$ - calculated (simulated) water depth, $\Delta\, h_m$ - measured water depth.

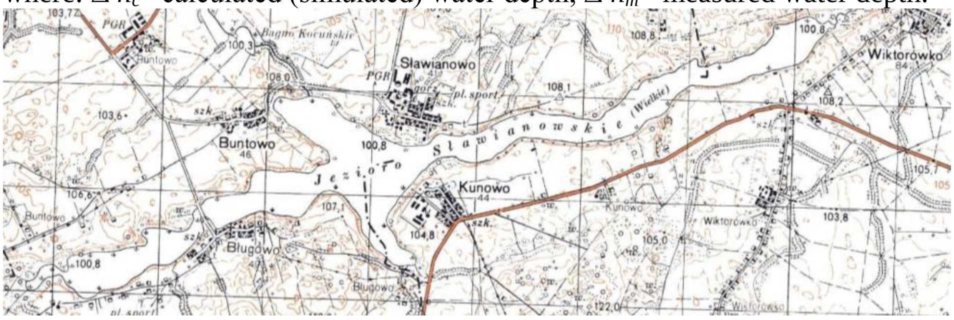

Fig. 1. Topographic map of the Sławianowskie Lake. Distances between coordinate lines are equal to 1 km. (source: Geoportal 2)

The basic parameter, tuned during model calibration, was the averaged hydraulic conductivity *k* in Eq. (2.6). Due to the lack of information on piezometric heads of ground water surrounding the lake, the *H* values for successive months were taken as the yearly mean water depths (measured from the mean lake bottom level) and - in an alternative approach - the depths at the end of the previous month with an initial value for the first month (November) *H* = 5.0 m and 7.7 m denotations used in the formulae should be explained.

4. PRELIMINARY RESULTS AND DISCUSSION

Simulations were carried out for constant lake surface area A = 276 ha consisted of the two above mentioned parts: the shallower one of area A_1 = 112.7 ha and the deeper one of area A_2 = 163.3 ha. The lengths of the bank line boundaries were taken as L_{B1} = 9920 m and L_{B2} = 11705 m, respectively. The distance between bank line and the point of measuring H along a transect was taken arbitrary as L =150 m.

Results of calculations for groundwater heads H equal to the lake water depths h at the end of the previous month, are shown in Table 1.

Table 1. Water balance of the Sławianowskie Lake in the hydrological year 2009 (all values in mm)

Month	*P*	*E*	Stream inflow	Stream outflow	Underground inflow/outflow		Storage Δh		
							measured	calculated	
					1 basin	2 basins		1 basin	2 basins
Nov.	44	-6	313	-83	-265	-265	60	58	59
Dec.	35	-3	313	-129	-211	-214	100	127	48
Jan.	25	-5	10	-301	265	265	-130	-112	-116
Feb.	38	-12	36	-82	17	17	50	-57	-55
Mar.	61	-57	115	-153	36	37	80	51	56
Apr.	3	-62	149	-268	176	175	-170	-32	-41
May	82	-48	27	-167	15	16	10	4	6
Jun.	94	-66	149	-303	28	28	-60	31	37
Jul.	71	-112	150	-301	-18	-18	-80	8	0
Aug.	31	-66	54	-215	72	72	-100	-9	-10
Sep.	43	-10	1	-151	26	26	-40	-9	-10
Oct.	70	-9	299	-339	-324	-323	30	84	87
Year	**597**	**-456**	**1616**	**-1392**	**-183**	**-184**	**-250**	**144**	**61**

Evaporation from the lake surface (304 mm·year^{-1}) calculated using Eq. (2.2) has occurred relatively small - most probably due to omitting transpiration of plants. Prządka [11] stated - on the base of investigations on small lake Łękuk - that the Penman formula underestimates the evaporation at least by 20%. Dąbrowski found that the evaporation from the Rajgrodzkie lake from May to October 2005 calculated by the Penman method reached 621 mm [2]. The evaporation values calculated by the Penman formula (2.2) were therefore multiplied by coefficient 1.5 giving yearly 456 mm. The results after such correction were close to those measured at the IMGW evaporation station in Piła.

Taking into account the clogging layer of sediments the value of the calibrated parameter (averaged hydraulic conductivity) k = 445 m·d^{-1} has occurred relatively high, but one should remember that it reflects also the impact of diffuse runoff on the underground inflow.

As can be concluded from an analysis of data in Tab. 1, the storage prediction accuracy is equal or less than 14 cm·month^{-1} (on average ±6 cm·month^{-1}, but during the whole year 2009 as high as 31-39 cm·year^{-1}), what can be acceptable for some practical purposes, but generally it is still unsatisfactory. The maximum errors have occurred in February and April, which may be connected with snow accumulation and melting. Anyhow, further studies are needed to improve the model.

5. CONCLUSIONS

Non-linear interrelations between water balance elements and lake water depth create errors in calculating their values using long time intervals, thus the balancing time interval should be as short as possible (1 day) and the segments of the lake as uniform as reasonable to minimize the errors.

The Penman formula (2.2), even without reduction factor suggested later by the author, has underestimated evaporation from the sławianowskie lake, most probably due to omitting evapotranspiration by plants.

To estimate properly groundwater inflow/outflow to/from a lake, periodic field measurements of piezometric heads in piezometers or wells located around the lake are indispensable.

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MODELOWANIE BILANSU WODNEGO PIĘTRZONYCH JEZIOR ZA POMOCĄ PROGRAMU KOMPUTEROWEGO MATLAB-SIMULINK®

Streszczenie

Praca przedstawia bilans wodny przepływowego piętrzonego jeziora, uwzględniający dopływ powierzchniowy, dopływ i odpływ podziemny opisany równaniem Dupuita, opad na powierzchnię jeziora, parowanie z powierzchni wody oraz odpływ w przekroju zamkniętym jazem piętrzącym. Z uwagi na nieliniowe związki wymienionych składników bilansu z poziomem wody w jeziorze, do obliczeń wykorzystano program komuterowy Matlab-Simulink®. Przydatność modelu sprawdzono na przykładzie Jeziora Sławianowskiego o powierzchni 276 ha i średniej głębokości - 6,6 m. Jezioro to zostało podzielone na dwa akweny o zróżnicowanej głębokości. Wyniki obliczeń miesięcznych bilansów wodnych dla roku hydrologicznego 2009, wykazały dobrą zgodność z pomiarami jedynie dla trzech pierwszych miesięcy. Stwierdzono, że dla zmniejszenia błędów obliczeniowych należałoby skrócić interwał bilansowania do jednej doby. Kalibracja modelu byłaby łatwiejsza i bardziej adekwatna, gdyby do oszacowania przewodności hydraulicznej przyległych do jeziora gruntów i osadów dennych wykorzystać badania poziomów wody w piezometrach, zlokalizowanych w kilku transektach, prostopadłych do linii brzegowej jeziora.

Słowa kluczowe: bilans wodny, podpiętrzone jezioro, Matlab-Simulink®, Jezioro Sławianowskie

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