Analytical solutions of a routing problem for storm water in a detention basin

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Abstract Analytical solutions of a routing problem for storm water flowing through a linear reservoir are presented for the assumption of trapezoidal-shaped inflow hydrograph. The maximum ponded (water) depth in the detention basin is chosen as a main design criterion. Calculations are carried out for a given rain recurrence interval but for various rain durations and sand filter surface areas to reach the maximum permitted ponded depth. A design example is also provided.

Key words detention basin; sand filter; storm water

Solutions analytiques d'un problème de transfert d'écoulement pluvial à travers un bassin de rétention

Résumé L'article présente des résolutions analytiques du problème de la transformation de l'écoulement à travers un réservoir de type linéaire, en supposant une forme trapézoïdale de l'hydrogramme entrant. Le critère principal de dimensionnement du bassin de rétention est la profondeur admissible de l'eau dans le réservoir. Les calculs ont été effectués en considérant un intervalle de récurrence des pluies donné, mais pour différentes durées de pluie et différentes surfaces du filtre à sable, afin d'estimer la profondeur maximale. Un exemple de dimensionnement est présenté.

Mots clefs bassin de rétention; filtre à sable; eaux pluviales

INTRODUCTION

Sand and gravel filters are recently utilised in management of storm water and wastewater from combined sewers. By connecting with detention basins they create good treatment facilities and hydraulic breaks for storm flows. Thus, they purify the polluted storm- and wastewater and diminish flooding risk in the receiving sewers and water bodies. There are no reliable methods of designing such connected facilities (Urbonas, 1997; Kasting, 2000). Both quantity and quality of storm water are normally uncertain. Suspended solids in the storm water can clog sand pores in the filter upper-layers and diminish their permeability.

The purpose of the paper is to present an analytical solution of a routing problem which can be helpful in designing storm water detention and filtration facilities. The proposed solution disregards clogging; however, that phenomenon can be also included in the calculation procedure. In the last case the main problem is providing reliable input data, which are highly changeable and dependent on rain characteristics and local conditions.

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Similar results can be obtained using numerical procedures; however, analytical formulae are easier in practical usage and—as exact solutions—can serve as a control of correctness of the numerical solutions.

STATEMENT OF THE PROBLEM

There are several considerations involved in the design of storm water detention and filtration facilities. These are (Chow *et al.*, 1988):

- the selection of design rainfall event,
- the volume of storage needed,
- the maximum permitted release rate,
- pollution control requirements and opportunities, and
- design of the outlet works for releasing the detained storm water or wastewater.

For hydraulic calculations of a detention basin one can use flow simulation models such as the HEC-1 model to perform reservoir routing. Mathematical description of the problem is based on the continuity equation:

$$\frac{dS}{dt} = Q(t) - Q_f(t) \tag{1}$$

where S is storage, $Q(t)$ is inflow, $Q_f(t)$ is outflow through the sand filter, and t is time.

For a typical detention basin with sloped walls (Fig. 1) and a laminar seepage flow through the filter, equation (1) can be rewritten as:

$$L(B + 2mh)\frac{dh}{dt} = Q(t) - k\frac{h + h_f}{h_f} \cdot B \cdot L \tag{2}$$

where L is the length of the basin, B is the width of the basin, m is the side (bank) slope, h is the ponded depth, k is the permeability of the filter medium, and h_f is the thickness of the filter bed.

When the water is flowing through a sand filter, the output rate is proportional to the ponded depth h. A detention basin with vertical walls ($m = 0$) will then be a linear reservoir, and with sloped walls—a nonlinear one. The nonlinearity is caused by the depth h within the left-hand side parentheses.

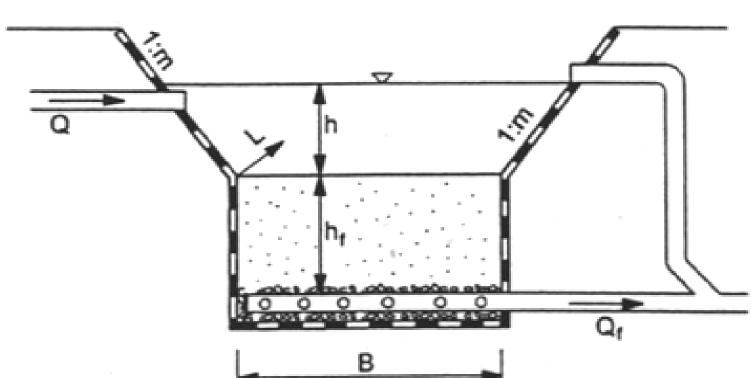

Fig. 1 Cross-section of the detention basin and sand filter.

There are many rainfall–runoff hydrological models. One of the simplest is a modified rational method, often used for sewer design (Chow *et al.*, 1988; Wanielista *et al.*, 1997). The maximum design flow is expressed by the formula:

$$Q_p = C \cdot i \cdot A \tag{3}$$

where Q_p is peak discharge, C is the runoff coefficient, i is rainfall intensity, and A is the watershed area.

Donahue *et al.* (1981) elaborated a simplified detention reservoir design method assuming that both inflow and outflow hydrographs have trapezoidal forms.

In the present paper, a trapezoidal-shaped hydrograph for inflow only is assumed and the shape of the outflow hydrograph is determined by calculation. It is assumed that the storm water inflow rate and ponded depths are changing in time, as shown in Table 1.

Table 1 Ranges of inflow rate and ponded depth in time.

Phase	Interval of time t	Inflow rate $Q(t)$	Ponded depth $h(t)$
I	0–t_c	$Q_p \cdot t/t_c$	$0 \to h_1$
II	t_c–D	Q_p	$h_1 \to h_2$
III	D–$D + t_c$	$Q_p(1-(t-D)/t_c)$	$h_2 \to h_m \to h_3$
IV	$>(D + t_c)$	0	$h_3 \to 0$

t_c: concentration time, Q_p: peak inflow rate, h_1: ponded depth at $t = t_c$ (Fig. 2), h_2: ponded depth at $t = D$, h_m: maximum ponded depth, h_3: ponded depth at $t = D + t_c$.

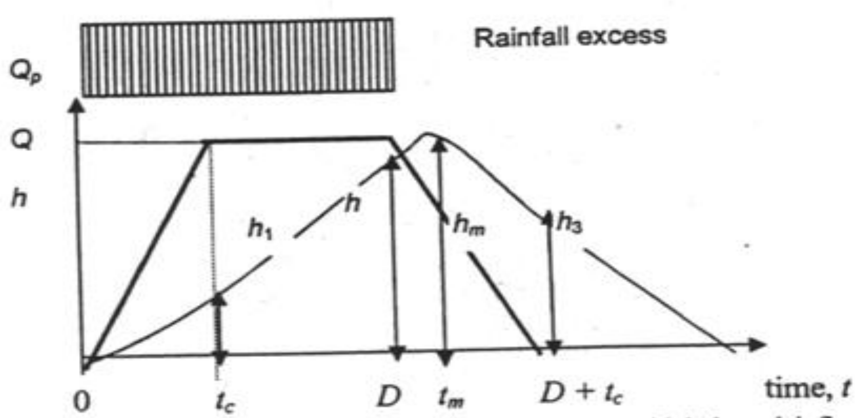

Fig. 2 Assumed rainfall excess ($CiDA$), trapezoidal-shaped inflow hydrograph $Q(t)$ and calculated ponded depths $h(t)$.

To solve the differential equation (2), the following additional simplifying assumptions are made:

- the shorter (end) walls of the basin are vertical,
- sand filter is saturated at the beginning of the storm,
- permeability of the sand is constant in space and time, i. e. there is no clogging of the filter medium, and
- the sand filter area A_f is negligibly small in comparison with the watershed area A.

SOLUTION OF THE PROBLEM

An analytical solution of the problem is possible for $m = 0$, i.e. when the detention basin as well as the sand filter have vertical walls embracing area A_f. This is the case of a linear reservoir created by the detention basin.

Phase I: for $0 \le t \le t_c$

$$h = \frac{h_f \cdot Q_p}{k \cdot t_c \cdot A_f}\left\{t - \frac{h_f}{k}\left[1 - \exp\left(-\frac{k \cdot t}{h_f}\right)\right]\right\} - h_f\left[1 - \exp\left(-\frac{k \cdot t}{h_f}\right)\right] \tag{4}$$

where Q_p is the peak inflow rate, and t_c is concentration time.

Phase II: for $t_c \le t \le D$

$$h = h_1 \cdot \exp\left[-\frac{k \cdot (t - t_c)}{h_f}\right] - \frac{h_f \cdot (Q_p - k \cdot A_f)}{k \cdot A_f}\left\{\exp\left[-\frac{k \cdot (t - t_c)}{h_f}\right] - 1\right\} \tag{5}$$

where h_1 is the ponded depth at $t = t_c$ calculated by equation (4).

Phase III: for $D \le t \le D + t_c$

$$h = \frac{h_f \cdot Q_p}{k \cdot t_c \cdot A_f}\left[D - t + \left(\frac{h_f}{k} + t_c\right)(1 - e^*)\right] + h_f(e^* - 1) + h_2 e^* \tag{6}$$

where $e^* = \exp(k(D - t)/h_f)$, and h_2 is the ponded depth at $t = D$ calculated by equation (5).

Comparing the derivative dh/dt to 0 one can obtain the time duration to achieve the maximum ponded depth:

$$t_m = D - \frac{h_f}{k}\left\{\ln h_f - \ln\left[(h_f + kt_c) - \frac{h_f Q_p}{t_c A_f}(h_2 + h_f)\right]\right\} \tag{7}$$

and the maximum value of the ponded depth itself:

$$h_m = \frac{h_f Q_p}{k \cdot t_c \cdot A_f}\left\{D - t_m + \frac{h_f}{k} + t_c\left[1 - \exp\frac{k(D - t_m)}{h_f}\right]\right\} - h_f + (h_2 + h_f)\exp\frac{k(D - t_m)}{h_f} \tag{8}$$

Phase IV: for $t > D + t_c$ when $Q = 0$:

$$h = (h_3 + h_f) \cdot \exp\left[\frac{k(D + t_c - t)}{h_f}\right] - h_f \tag{9}$$

where h_3 is the ponded depth at $t = D + t_c$ calculated by equation (6).

When the detention basin walls are sloped ($m > 0$) an analytical solution is no more obtainable, thus a numerical integration of the continuity equation (2) is needed.

Knowing the ponded depth h from equations (4)–(6) and (8)–(9), one can calculate the outflow rate (which is equal to the seepage flow rate) according to equation (2):

$$Q_f(t) = k\frac{h+h_f}{h_f}A_f \tag{10}$$

Relationship (10) is valid for both vertical and sloped basin walls.

For a given recurrence interval of flooding (or overflow) the calculations should be repeated for various values of sand filter surface area and rain duration to find the maximum ponded depth h_m close to the permitted one. Another design criterion would be the maximum permitted release rate Q_f (*Chow et al.*, 1988).

DISCUSSION

Our analysis showed that, even in linear detention reservoirs, the trapezoidal inflow hydrograph may be transformed into an outflow hydrograph of irregular shape. That result shakes the assumption made by Donahue *et al.* (1981) that an outflow hydrograph can be also a trapezoidal one. It would be possible in some cases, e.g. when the filter outflow is equal to the peak inflow (steady-state conditions), but not smaller than the peak inflow, as assumed the above-mentioned authors.

The maximum permitted ponded depth h_m is typically taken in practice as 1.0 m (Kasting, 2000). Above that level, an overflow pipe (see Figure 1) or spillway is installed to control the ponded depth. Sandy soils with the permeability coefficient $k = 10^{-4}$ to 10^{-5} m s^{-1} are recommended for an effective treatment. The filter depth h_f is equal to 0.8 to 1.2 m. Its surface is often vegetated by reed (*Phragmites australis*) or other wetland plants. Good mechanical pre-treatment with removal of fine mineral particles and organic matter is needed to avoid clogging of the filter medium. This especially concerns combined sewer overflows which frequently carry a lot of organic matter. Safe daily loads of volatile suspended solids are 5 g m^{-2}day^{-1} for sands, and up to 40 g m^{-2}day^{-1} for gravel (Bavor & Schulz, 1993). Another problem is mechanical clogging by small mineral particles (Urbonas, 1997).

The German experience shows that yearly hydraulic loads can reach 20 or even 37 m^3 m^{-2} year^{-1} (Kasting, 2000). The maximum outflow (release) rate depends on the sewer capacity and typically lies between 10 and 15 dm^3 s^{-1}. The full capacity of a pipe of inner diameter of 250 mm, laid on a minimum gradient of 0.3%, is equal to 33 dm^3 s^{-1}.

The treatment efficiency varies for BOD_5 in the range 36–87% and similarly for suspended solids: 38–94% (Kasting, 2000).

To illustrate the calculation procedure, an example is given below.

Example

Calculate surface area A_f of detention basin and sand filter to drain a watershed of area $A = 20\,000$ m^2 (2 ha), for which the runoff coefficient $C = 0.5$. Concentration time t_c = 2.0 min. Assume that the basin walls are vertical, the maximum permissible ponded depth $h_m = 1.0$ m, the thickness of the filter bed $h_f = 0.8$ m and the filter medium permeability $k = 10^{-4}$ m s^{-1}. The facility should withstand the 10-year recurrence interval rain. Estimate the maximum outlet flow rate.

Solution

Calculations were made using equations (4)–(10). To estimate the peak discharge Q_p, an intensity–duration–frequency rain curve is needed. One such relationship, relevant to lowland areas in Poland, is Błaszczyk's formula, based on Gorbachev's equation, in the following form (Błaszczyk *et al.*, 1983):

$$i = \frac{\alpha \cdot \sqrt[3]{H^2 T_r}}{D^{0.67}} \tag{11}$$

where i is rainfall intensity (dm^3 s^{-1} ha^{-1}); α is climatic coefficient (dimensionless); H is long-term annual average precipitation (mm year^{-1}); T_r is the recurrence interval, i.e. rain return period (years); and D is the rainfall duration (min).

Rainfall intensities and corresponding peak discharges were calculated for $\alpha = 6.631$, $H = 600$ mm year^{-1}, $T_r = 10$ years and various rain durations, D. Various values of the sand filter area $A_f = L \cdot B$ were taken into account. It occurred that the design condition $h_m \approx 1.0$ m was met by $A_f = 150$ m^2. For those data the outlet flow rate $Q_f = 33.2$ dm^3 s^{-1}. The calculation results are shown in Fig. 3. It can be seen that the critical rain duration is equal to 60 min. Calculations made for maximum yearly rains (T_r = 1 year) and $h_m = 1.0$ m showed that for the watershed reduced (C = 1) surface area A = 1.0 ha, the corresponding sand filter areas were equal to A_f = 66 m^2 ha^{-1} at $k = 10^{-4}$ m s^{-1} and A_f = 145 m^2 ha^{-1} at $k = 10^{-5}$ m s^{-1}. The influence of the concentration time on A_f in these cases was negligible.

It is possible to take into account clogging phenomena (Błażejewski & Murat-Błażejewska, 1997). For that purpose concentration of suspended solids in the inflowing water should be known. Organic matter can be degraded in time, especially under aerobic conditions, restoring the hydraulic permeability. However, fine mineral particles (silt and clay) may clog the surface layer irreversibly. In such a case, the simplest method of recovering the filter permeability is removal of the surface layer of depth 1–3 cm.

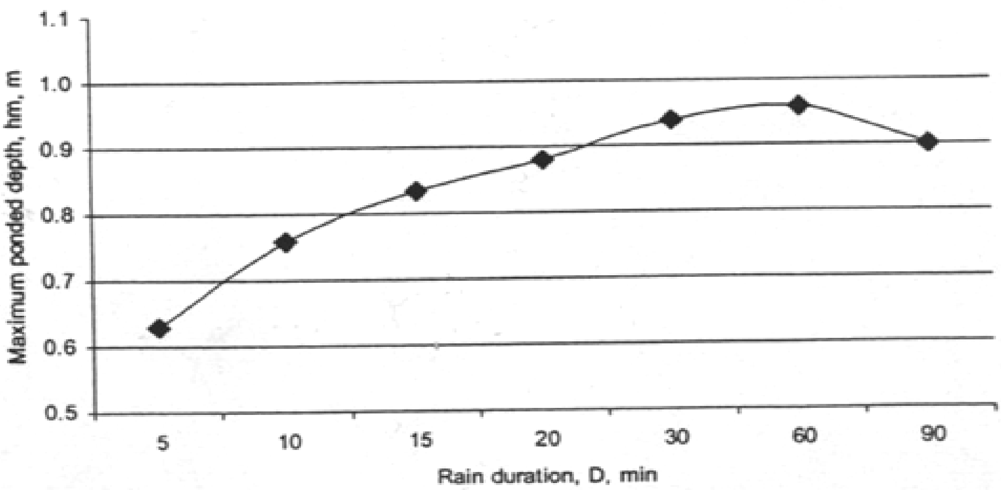

Fig. 3 Maximum ponded depths h_m calculated by way of example.

CONCLUSIONS

- Sand and vegetated soil filters are good hydraulic breaks and treatment facilities for both storm water and combined sewer overflows.
- Detention basins and soil filters with vertical walls can be calculated analytically (equations (3)–(10)) and the basins with sloped walls, as nonlinear reservoirs—numerically.
- For a given recurrence interval of flooding (or overflow) the calculations should be repeated for various values of sand filter surface area and rain duration to find the maximum ponded depth close to the permitted one. Another design criterion would be the maximum permitted release rate.
- Good mechanical pre-treatment of the wastewater is recommended to avoid filter clogging. The clogging phenomena can be also included into the design procedure.

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Received 14 March 2002; accepted 19 February 2003